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Model equations for the dispersion of the input function in bolus infusion PET studies

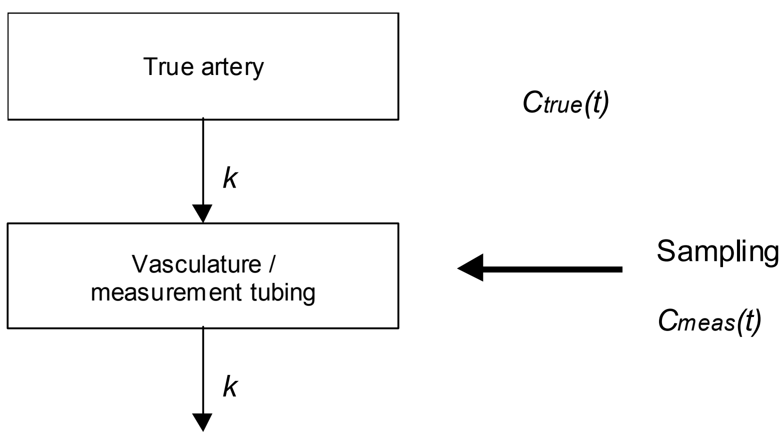
Iida et al. [1] suggested that a single monoexponential function *d(t)* could be used to represent the effect of dispersion,

$$d(t) = \frac{1}{\tau} e^{-t/\tau} \qquad (1)$$

where τ denotes the time constant of the dispersion. The measured arterial curve $C_{meas}(t)$ and the true arterial curve $C_{true}(t)$ are related as

$$C_{meas}(t) = C_{true}(t) \otimes d(t) \qquad (2)$$

This dispersion in the arterial line can be thought of been caused by a compartment through which the blood must flow before it is measured (measurement tubing or arterial vasculature). The rate constants from and to the compartment are $k=1/\tau$:



Then the relations between the true and measured concentrations in the compartmental model are as given in Eq. (3):

$$C_{meas}(T) = k\int_0^T C_{true}(t)dt - k\int_0^T C_{meas}(t)dt \qquad (3)$$

By applying linear interpolation and Kuwabara's approach for partial solution of the differential equations [2], the integrals of true and measured radioactivity concentrations can be presented as in Eqs. (4) and (5), where Δt is the frame length:

$$\int_0^T C_{true}(t)dt = \int_0^{T-\Delta t} C_{true}(t)dt + \frac{\Delta t}{2} C_{true}(T-\Delta t) + \frac{\Delta t}{2} C_{true}(T) \qquad (4)$$

$$\int_0^T C_{meas}(t)dt = \int_0^{T-\Delta t} C_{meas}(t)dt + \frac{\Delta t}{2} C_{meas}(T-\Delta t) + \frac{\Delta t}{2} C_{meas}(T) \qquad (5)$$

Eq. (5) can be substituted in Eq. (3) to give Eq. (6), which can be used to simulate the effect of dispersion:

$$C_{meas}(T) = \frac{k}{1 + k\frac{\Delta t}{2}}\left(\int_0^T C_{true}(t)dt - \int_0^{T-\Delta t} C_{meas}(t)dt - \frac{\Delta t}{2}C_{meas}(T - \Delta t)\right) \qquad (6)$$

In contrast, to remove (correct) the effect of dispersion from the measured data, the integral of true arterial curve can first be solved and calculated from Eq. (3), and after that the true curve can be calculated by reorganization of Eq. (4):

$$\int_0^T C_{true}(t)dt = \frac{1}{k}C_{meas}(T) + \int_0^T C_{meas}(t)dt \qquad (7)$$

$$C_{true}(T) = \frac{\int_0^T C_{true}(t)dt - \int_0^{T-\Delta t} C_{true}(t)dt}{\left(\Delta t/2\right)} - C_{true}(T - \Delta t) \qquad (8)$$

In practice, Eq. (8) may give very noisy data, and thus the curves need to be smoothed. One method to do that is to calculate curve from Eq. (9) instead:

$$C_{true}(T) = \frac{\int_0^{T+\Delta t} C_{true}(t)dt - \int_0^{T-\Delta t} C_{true}(t)dt}{2\Delta t} \qquad (9)$$

References

1. Iida H, Kanno I, Miura S, Murakami M, Takahashi K, Uemura K. Error analysis of a quantitative cerebral blood flow measurement using $H_2{}^{15}O$ autoradiography and positron emission tomography, with respect to the dispersion of the input function. *J. Cereb. Blood Flow Metabol.* 1986; 6:536-545.
2. Kuwabara H, Cumming P, Reith J, Léger G, Diksic M, Evans AC, Gjedde A. Human striatal L-DOPA decarboxylase activity estimated in vivo using 6-[^{18}F]fluoro-DOPA and positron emission tomography: error analysis and application to normal subjects. *J. Cereb. Blood Flow Metab.* 1993; 13:43-56.