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Model equations for [^{15}O]H_2O PET perfusion (blood flow) studies

The traditional two-compartmental blood flow model is based on two equations, defining the concentration of labeled water in tissue, $C_T(t)$, and the concentration in tissue region including the vascular radioactivity, $C_{PET}(t)$, as a function of time, t:

$$C_T(T) = K_1 \int_0^T C_A(t)dt - k_2 \int_0^T C_T(t)dt \quad (1)$$

$$C_{PET}(T) = V_A C_A(T) + C_T(T) \quad (2)$$

$C_A(t)$ represents the arterial radioactivity concentration, V_A is the arterial volume fraction in tissue, K_1 is the tracer rate constant from blood to tissue, corresponding to blood flow, f, in this model, and k_2 is the rate constant for transfer back to blood. K_1/k_2 represents the partition coefficient of water, p, between tissue and blood. Venous radioactivity concentration is assumed to be equal to the concentration in tissue.

By applying linear interpolation and Kuwabara's approach for partial solution of the differential equations [1], the integral of tissue radioactivity can be presented as in Eq. (3), where Δt is the frame length:

$$\int_0^T C_T(t)dt = \int_0^{T-\Delta t} C_T(t)dt + \frac{\Delta t}{2} C_T(T-\Delta t) + \frac{\Delta t}{2} C_T(T) \quad (3)$$

Eq. (3) can be substituted in Eq (1) to give Eq. (4), after which the regional time-radioactivity curve can be calculated using that and Eq. (2):

$$C_T(T) = \frac{K_1 \int_0^T C_A(t)dt - k_2 \left[\int_0^{T-\Delta t} C_T(t)dt + \frac{\Delta t}{2} C_T(T-\Delta t) \right]}{1 + k_2 \frac{\Delta t}{2}} \quad (4)$$

Another possibility is to solve the regional time-radioactivity curve directly from the previously given equations. First, $C_T(t)$ is solved from Eq. (2), and the resulting equation (5) is then integrated to give Eq. (6):

$$C_T(T) = C_{PET}(T) - V_A C_A(T) \quad (5)$$

$$\int_0^T C_T(t)dt = \int_0^T C_{PET}(t)dt - V_A \int_0^T C_A(t)dt \quad (6)$$

These are then substituted in Eq. (1), giving Eq. (7):

$$C_{PET}(T) = V_A C_A(T) + (K_1 + V_A k_2) \int_0^T C_A(t)dt - k_2 \int_0^T C_{PET}(t)dt \quad (7)$$

This could be used to solve the model parameters with multilinear regression analysis, as first proposed by Blomqvist [2]. By applying linear interpolation, as was done in Eq. (3), we get Eq. (8), suitable for simulation and non-linear regression analysis:

$$C_{PET}(T) = \frac{V_A C_A(T) + (K_1 + V_A k_2)\int_0^T C_A(t)dt - k_2\left[\int_0^{T-\Delta t} C_{PET}(t)dt + \frac{\Delta t}{2} C_{PET}(T-\Delta t)\right]}{1 + k_2 \frac{\Delta t}{2}} \quad (8)$$

References

1. Kuwabara H, Cumming P, Reith J, Léger G, Diksic M, Evans AC, Gjedde A. Human striatal L-DOPA decarboxylase activity estimated in vivo using 6-[^{18}F]fluoro-DOPA and positron emission tomography: error analysis and application to normal subjects. *J. Cereb. Blood Flow Metab.* 1993; 13:43-56.
2. Blomqvist G. On the construction of functional maps in positron emission tomography. *J. Cereb. Blood Flow Metab.* 1984; 4:629-632.