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# Model averaging with Akaike weights

This document reviews the method for using Akaike information criteria (AIC) as weights in averaging parameters of models of different complexity [Turkheimer et al., 2003].

## Computation of AIC

The traditional formulation of Akaike Information Coefficient (AIC) is [Turkheimer et al., 2003]

$$AIC = -2\ln L(\theta) + 2k, \tag{1}$$

where $L(\theta)$ is the maximized likelihood value [Forster 2000] and $k$ is the number of estimable parameters in the model.
When $k$ is large relative to $n$ ($n/k < 40$), it is recommended to use bias-adjustment [Turkheimer et al., 2003]:

$$AIC = -2\ln L(\theta) + 2k + \frac{2k(k+1)}{n-k-1}. \tag{2}$$

The bias-adjustment term becomes very small when the size of *n* increases relative to *k*. So there is no harm using it even when the condition is not met.
In the special case of sum of squares optimization, the basic AIC formula is expressed as [Turkheimer et al., 2003]

$$AIC = n\ln(\hat{\sigma}^2) + 2k, \tag{3}$$

where $\hat{\sigma}^2 = \frac{\sum \hat{\varepsilon}_i^2}{n}$. Here $n$ is the sample size and $\hat{\varepsilon}_i$'s are the estimated residuals. If variances are unequal in different time points *i*, then weighted residuals should be used in place of $\hat{\varepsilon}_i$. The bias-adjustment term is used similarly to equation (2).

From the AIC values calculated for different models, one can see which model fits the best for the given data set. **The best model is the one with lowest AIC value**.
Absolute AIC values should never be analyzed because they only have meaning when compared between models fitted for a given data set.

## Computation of model weights

For the computation of Akaike weights let's define

$$\Delta_i = AIC_i - \min AIC, \tag{4}$$

where min *AIC* is the smallest value of *AIC* in the model set. Akaike weights are now calculated from the formula [Turkheimer et al., 2003]

$$w_i = \frac{\exp(-\Delta_i / 2)}{\sum_{r=1}^{M} \exp(-\Delta_r / 2)},$$

where *M* is the number of models.

A comparison between AIC and F-test has been published by Giatting et al 2007. They conclude that AIC is effective and efficient approach. Compared to F-test, it has the advantage of being suited both for nested and non-nested models. Giatting et al. also note that F-test tends to choose more complex models.

## References


1. Turkheimer, Hinz, Cunningham: *On the undecidability among kinetic models: from model selection to model averaging.* Journal of Cerebral Blood Flow & Metabolism, 23:490-498, 2003
2. Forster: *Key Concepts in Model Selection: Performance and Generalizability* Journal of Mathematical Psychology 44, 205-231, 2000
3. Giatting G, Gletting P, Reske S.N, Hohl K, Ring C: *Choosing the optimal fit function: Comparison of the Akaike information criterion and the F-test.* Med. Phys. 34 (11): 4285-92, 2007