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Applications of wavelets in PET modelling

Introduction

This document reviews the applications of wavelets used in literature. In Hammersmith hospital in London Federico Turkheimer with his co-workers have published a trilogy of articles [Turkheimer et al. 1999], [Turkheimer, Banati et al. 2000], [Turkheimer, Brett et al. 2000] concerning the use of wavelets in emission tomography. They have used wavelets in reconstruction of the dynamic image and construction of parametric images and maps.
In Stockholm's Karolinska hospital a group of researchers has studied wavelets. In their article [Cselenyi et al. 2002] wavelet analysis has been compared with two other modelling methods.

London

I
In the first article of the trilogy [Turkheimer et al. 1999] Fourier transform is stated to be unsuitable for nonstationary signals because all the time information is lost when signal is estimated with only different frequency components. Wavelet transform is considered suitable for this purpose, because it takes into account also local frequency changes. The theoretical framework of wavelets is established and two versions of wavelet transforms, dyadic wavelet transform (DVT) and translation invariant dyadic wavelet transform (DWT-TI) are demonstrated. DWT-TI is translation invariant because it averages over all transformations that are received when using basic DWT.

Choice of the optimal wavelet space is also considered. It is stated that the base should be orthogonal to preserve the noise attributes of the spatial domain and that wavelet function should be symmetric to avoid phase distortions. Wavelet transform should then minimize the correlations between wavelet coefficients at different resolutions.

Choice of the number *n* of wavelets should be a compromise: the smaller polynomial degree the sharper wavelets (they represent better local discontinuities of the signal) where as greater polynomial degree reduces correlations between different resolutions.

Thresholding is done to prevent negligible coefficients to appear. *Universal thresholding* of Donohu and Johnstone (1994) is said to ensure a noise-free reconstruction but tends to under fit the data. *Bonferroni* correction is based on the idea that all coefficients are generated by a null normal distribution. *Level-wise chi-square* testing makes sure every resolution level is significant and zeroes the nonsignificant coefficients.

The denoising of dynamic images was first applied to a phantom study (Hoffman brain phantom). A single slice was selected and used with both DWT and DWT-TI analysis. With DWT some artefacts were found in the treated image but in the DWT-TI treated version they were all gone.

Two thresholding policies were compared with an FDG brain study. Universal thresholding was notified to give sharper images while SURE thresholding detected details at the finest resolution.
In a [^{11}C]PK11195 study of peripheral benzodiazepine receptors in brain, the SURE filter was considered to work very well.

II
The second article [Turkheimer, Banati et al. 2000] introduces wavelets as means to get better parametric images from dynamic PET images. It describes a process, where all the kinetic analysis is done in the wavelet domain. The estimation of a parametric image is implemented with four steps: first the DWT is applied to the original dynamic image, then kinetic modelling is done in the wavelet domain and after thresholding the inverse wavelet transform is applied.

For validation purposes this technique was first used with artificial dataset with nonstationary noise field. An FDG study was performed with Patlak approach to the kinetic modelling. The parametric images were found mostly denoised and fine details preserved. Some artefacts appeared as dots, but they could be eliminated with pooling.
[^{11}C]raclopride dynamic study estimated D2-receptor distribution in brain. Direct application of Logan plot was compared to the wavelet analysis approach. Parametric maps received with direct application were discovered corrupted by high levels of noise, but with wavelet approach the noise was completely removed.

III
In the third article [Turkheimer, Brett et al. 2000] a new procedure is introduced for getting statistical maps from multiple scans. The scans can be from different subjects or measured in different conditions or in repeated occasions. With this approach all the statistical analysis is done in wavelet space as in the case of dynamic image all the kinetic analysis was done in wavelet space.

The technique was validated by a randomisation study. Null dataset was composed with a activation/rest -study, where the subject looked at a computer monitor, and was told either to react to the stimuli or just to watch without moving. H_2O^{15} was used to detect cerebral blood flow. Two groups of simulated datasets were constructed from the null dataset and statistical analysis was performed on each dataset.
After validation, method was used with another H_2O^{15} flow study and a serotonin receptor study. Parametric study of cerebral blood flow response to word recognition was implemented with five right-handed normal subjects, who viewed single words presented at different rates per minute.
In the receptor study the effect of depression on brain serotonin receptors was studied with [^{11}C]WAY-100635. Use of statistical parametric mapping (SPM) and wavelet analysis was compared. WT analysis was discovered to give more accurate results than SPM.

Stockholm

Article [Cselenyi et al. 2002] presents a comparison of four different modelling methods as part of a binding potential (BP) study with [^{11}C]FLB 457. Entire brain of ten healthy

mail subjects was scanned and the modelling was implemented with ROI-based analysis, pixel-by-pixel analysis and two versions of wavelet analysis.
Logans graphical analysis was used in the kinetic modelling in each four analysis approaches.
The ROI-based analysis was used as reference data. This means that for cross-validation purposes anatomical standardisation and averaging over ROI's has to be done with parametric images constructed with three other methods. The two wavelet-based methods were two-dimensional translation-invariant (2DTI) and three-dimensional (3-DWT) wavelet transform. Translation invariant property doesn't work in three dimensions because of the increasing computational requirements.
The binding potential values of pixel-by-pixel analysis were approximately 50% of those received with the ROI-based analysis in each brain region. Values from the wavelet approaches were in better agreement with the values of ROI-based analysis. The 2DTI method gave values that were approximately 78% of the reference values and with 3-DWT method the same values was approximately 100%. More homogeneous patterns were shown with 3-DWT than with 2DTI.

Conclusion

In all these studies the wavelets are used for denoising purposes. There has been predictions of wavelets' usefulness in many other applications also, but very little has been done in other research areas. All the articles agree on wavelets ability to model noise, but only very few ET centres really use wavelets in every day modelling.

References

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