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Using basis function method for parametric mapping

This document reviews the use of basis function method in parametric mapping. The method is at the moment applied for two cases: simplified reference tissue model (SRTM) for estimation of binding potential [Gunn et al 1997] and kinetic $H_2^{15}O$ model for estimation of blood flow.

SRTM model equations and basis functions

Simplified reference tissue model (SRTM) has been introduced in tpcmod0002. For the use of basis function method we review the operational equation of the model:

$$C_T(t) = R_I C_R(t) + \left(k_2 - \frac{R_I k_2}{1+BP}\right) C_R(t) \otimes e^{-[k_2/(1+BP)+\lambda]} \quad (1)$$

Here $C_T(t)$ is the concentration time course in the tissue of interest and $C_R(t)$ the concentration time course in reference tissue. R_I is local rate of delivery relative to rate of delivery in reference region and k_2 is the efflux rate constant from the tissue. λ is the physical decay constant of the isotope and $\otimes$ is the convolution operator.
Equation (1) can be rewritten as

$$C_T(t) = \theta_1 C_R(t) + \theta_2 C_R(t) \otimes e^{-\theta_3 t},$$

where $\theta_1 = R_I$, $\theta_2 = k_2 - R_I k_2/(1+BP)$ and $\theta_3 = k_2/(1+BP) + \lambda$.
This equation is linear in θ_1 and θ_2, which means that standard linear least square method can be used for estimation as long as value for θ_3 is fixed.
If we can choose bounds for the value of θ_3 then some number N of basis functions

$$B_i(t) = C_R(t) \otimes e^{-\theta_{3i} t}, \quad i \in (1, N)$$

can be formed.

Then for a basis function B_i equation

$$C_T(t) = \theta_1 C_R(t) + \theta_2 B_i(t)$$

is linear. In program *imgbfbp* QR decomposition (see tpcmod0025) was used to solve the linear least square problem for each basis function. Thus in order to get a parametric picture we have to do the estimation N times in each voxel and then choose the best result.

Gunn et al. suggest the bounds for parameter θ_3 to be

$\lambda < \theta_3^{min} \le \theta_{3_i} \le \theta_3^{max}$,

where $\theta_3^{min} \le k_2^{min} / (1 + BP^{max}) + \lambda$ and $\theta_3^{max} \ge k_2^{max} + \lambda$ and the value for N to be 100.

This application has been programmed in a C-program *imgbfbp*. In Appendix A results of this program are compared to results of two other programs *imgsrtm* and *imgdv*. All three are free software under the terms of the GNU LESSER PUBLIC LICENSE (LGPL) and can be downloaded from the webpage http://www.turkupetcentre.fi/staff/vesoik/programs/index.html .

$H_2^{15}O$ model equations and basis functions

Kudomi et al have presented the basis functions approach for $H_2^{15}O$ model [Kudomi et al 2009]. The traditional two-compartment model for $H_2^{15}O$ has been introduced in tpcmod0004. Operational equation according to Kudomi et al is:

$$C_i(t) = (1 - V_A) \cdot K_1 \cdot A_w(t) \otimes e^{-k2 \cdot t} + V_A A_w(t), \quad (2)$$

Where $C_i(t)$ is the radioactivity concentration is a PET image voxel, A_w is the arterial input, λ is the physical decay constant of the isotope and $\otimes$ is the convolution operator. The equation can be rewritten as

$$C_i(t) = \theta_1 \cdot B(k_2, t) + \theta_2 A_w(t),$$

Where $B(k_2, t) = A_w(t) \otimes e^{k_2 \cdot t}$ and

$\theta_1 = (1 - V_A) \cdot K_1$ and $\theta_2 = V_A$

Kudomi suggests physiologically reasonable range (0, 15.00) ml/min/g for k_2 and sufficient amount of discrete values in this range would be 1500 (?).

Pseudo algorithm - SRTM-basis function method example

To keep the basis function algorithm fast, as much computing as possible has to be done before the actual voxel loop. Here is the basic pseudo algorithm representing how parametric maps for SRTM can be formed with basis function method and QR decomposition:

for basis function B_i, i=1,…,N
    Calculate the QR decomposition of matrix $A = [C_R \; B_i]$ and save it
end for

for each voxel

for basis function B_i, i=1,…,*N*

compute and save estimates for parameters θ_1 and θ_2 from the problem $C_T = A\begin{bmatrix}\theta_1 \\ \theta_2\end{bmatrix}$,

where QR decomposition of A is already known

end for

select that basis function and the parameters corresponding to it that give the lowest value for the sum of squares $SS = \sum_{j=1}^{n} \left(C_{PET}(t_j) - C_T(t_j)\right)^2$

Count the binding potential $BP = \frac{k_2}{\theta_3 - \lambda} - 1$

end for

Other models

We haven't yet determined basis function applications for other models than the ones mentioned in this document. Model equations that contain two basis function are too heavy to compute. Possibilities of how to deal with these kinds of problems will be examined in the future.

References

1. Gunn, R. N., Lammertsma, A. A., Hume, S. P., Cunningham, V. J.: Parametric Imaging of Ligand-Receptor Binding in PET Using a Simplified Reference Region Model, *Neuroimage* 1997; 6:279-287.
2. Kudomi N, Koivuviita N, Liukko K, Oikonen V, Tolvanen T, Hidehiro I, Tertti R, Metsärinne K, Iozzo P, Nuutila P.: Parametric renal blood flow imaging using [^{15}O]H_2O and PET, *Eur J Nucl Med Mol Imaging* 2009; 36:683-691