Turku PET Centre Modelling report TPCMOD0036 2007-02-06
Kaisa Liukko

Comparing equality of two time activity curves with runs test

Introduction

This document reviews runs test –method as means to prove equality of two time activity curves (TAC).

Runs test –method

Runs test –method is based on the idea that independency of residuals of two curves indicates that the curves are statistically the same. It is assumed that the residuals should oscillate around the mean, which is close to zero. If we have data of N time points and the two curves are $y_1(t)$ and $y_2(t)$, the residual is then $res(t)= y_1(t)- y_2(t)$ in each time point.

Runs test method calculates the number of runs in the data. A run is defined as a sequence of residual having same sign. A small number of runs in the data indicate systematic error.

If R is the number of runs in a sequence of N residuals having n^- negative and n^+ positive values, then R is an outcome of random variable P that tends to normal distribution $N(\mu,\sigma^2)$, where

$$\mu = \frac{2n^+n^-}{N} + 1 \qquad \text{and} \qquad \sigma^2 = \frac{2n^+n^-(2n^+n^- - N)}{(N-1)N^2}.$$

Then a transformation of

$$Z = \frac{R - \mu}{\sigma}$$

gives us a random variable Z that tends to standardized normal distribution N(0,1).

According to a lower tail test, the residuals are then considered to be nonrandom (i.e. the assumption of independence of residuals is to be rejected) if at Z, the value of density function $\Phi(Z)$ is lower than given significance level i.e.

$$\Phi(Z) < \alpha$$

In the implementation of runs test in libtpcmodel_1_2_3, the default for significance level α is 5%.

References

1. Cobelli C, Forster D, Toffolo G. 2002 Tracer Kinetics in Biomedical Research: From Data to Model. Kluwer Academic Publishers.